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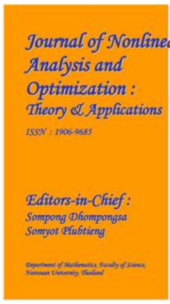

AN INVENTORY MODEL FOR A DETERIORATING ITEM WITH TRADE CREDIT POLICY AND ALLOWABLE SHORTAGES UNDER UNCERTAIN DEMAND

Mr. Vivek Mali Associate Professor Geetanjali Institute of Technical Studies, Dabok Udaipur,
Dr. Kunal Shah Head Department of Mathematics, SMMPISR, Kadi Sarva Vishwavidyalaya, Gandhinagar, Gujarat
Dr. Varun Humad Associate Professor Geetanjali Institute of Technical Studies, Dabok Udaipur, Rajasthan
Deepika Babel Assistant Professor Aravali Institute of Technical Studies, Umarda, Udaipur, Rajasthan

Abstract:
In this research an economic production quantity (EPQ) model with two level trade credit policy for a deteriorating good under fully backlogged shortages is formulated. In this model demand is characterized by trapezoidal fuzzy number. To verify model numerical examples are given. Sensitivity analysis has been carried out to identify the most critical parameters.

Introduction

The management of inventories is one of the serious responsibilities that the managers of manufacturing firms need to do carefully. Recently, businesses are highly competitive due to globalization. So, all manufacturing firms are highly engaged in how to promote their business in order to have a successful career with the aim of surviving in current volatile markets. For this reason, the researcher and academician are very interested in deriving inventory models that are useful in an inventory decision- making process.

In the inventory management, there exits two well-known inventory models: Harris (1913) and Taft (1918). The first one is Economic order quantity (EOQ). The second one is called the Economic Production quantity (EPQ). It is relevant to remark that the EOQ inventory model is particular case of the EPQ inventory model. Notice that the well-known EOQ inventory model is developed by putting the assumption that when a retailer buys a produce, he or she must give the payment to his/her supplier when the item is delivered.

For very small businesses and start-up companies, the trade credit (permissible Delay in period) plays a major role in inventory control for both the supplier as well as the Retailer. Approaching towards this concept of trade credit, Goyal (1985) framed an inventory model for the supplier offering permissible delay period to the buyer. Aggarwal and Jaggi (1995) extended Goyal's (1985) model for deteriorating items. In this direction the reader can see the two comprehensive reviews related to trade credit in Seifert et al. (2013).

Huang (2003) introduced an EOQ inventory model with two-level trade credit scheme by taking into consideration that the supplier gives to the retailer a delay period(M), and the latter correspondingly gives a delay period (N) to its customer. After that, this type of inventory model has also discussed by Teng (2009)

Mahata and Mahata (2011) investigated the economic order quantity bases inventory for a retailer under two levels of trade credit to reflect the supply chain management situation. Chung et al. (2014) developed an economic production quantity inventory model for deteriorating items under two levels of trade credit, in which the supplier offers to the retailer a permissible delay period and simultaneously the retailer in turn provides a maximal trade credit period to its customers in a supply chain system comprised of three stages. Ouyang et al. (2014) developed optimal credit period and lot size for deteriorating items with under two-level trade credit financing.

A major issue in any business transaction is that control and maintain the inventories of deteriorating items. Goods are deteriorating owing to their values go down with time. The deteriorated items cannot be repaired or replaced. Deterioration occurs due to evaporation, damage, spoilage, dryness, etc. and it reduces the quality and/or quantity of stored items. The inventory problem of deteriorating items was first studied by Whitin (1957), in which he proposed the fashion items deteriorating at the end of the storage period. After that Ghare and Schrader (1963) were the first to incorporate the idea of deterioration in inventory models. They studied an exponentially decaying inventory model with deterioration rate is known and constant. An economic order quantity inventory model for deteriorating items was developed by Bose et al. (1995). Rau H et al (2004) developed an inventory model for deteriorating items with a shortage occurring at the supplier involving a supply chain between the producer and buyer. Later, a lot of research works have been done under the trade credit policy (e.g. Liao et al. (2018), Shaikh et al. (2021), Soni and Shah (2021)).

Features of inventory management models are that the resulting optimal solutions can be implemented in a fast-changing situation where, for example, the conditions are changed daily. There is a need for new and effective methods for modelling systems associated with inventory management, in the face of uncertainty. Early works in using the fuzzy concept in decision making were done by Zadeh (1965) and Bellman (1970) through introducing fuzzy goals, costs, and constraints. EPQ model with different schemes of fuzzy input parameter have been proposed by Lee and Yao (1998).

Urgeletti (1983) treated EOQ model in fuzzy sense, and used triangular fuzzy number. Chen and Wang (1996) used trapezoidal fuzzy number to fuzzify the ordering cost, inventory cost, and backorder cost in the total cost of inventory model without backorder.Vujosevicetal. (1996) used trapezoidal fuzzy number to fuzzify the order cost in the total cost of inventory model with backorder. Yao & Lee (2003) considered the inventory model without backorder in which the order quantityis fuzzified as a triangular fuzzy number. Gani and Maheswari (2010) discussed the retailer’s ordering policy under two levels of delay payment considering the demand and selling price as triangular fuzzy number.Kao and Hsu (2002) considered a single-period inventory model with fuzzy demand. Hsieh (2002) analysed some production inventory models in fuzzy sense and he proposed some optimal strategies. Dutta and Pavan Kumar (2013) presented a fuzzy inventory model for deteriorating item in which rate of deterioration and demand are constant, shortages are allowed and fully backlogged. Iswarya and Karapagavalli (2021) developed fuzzy inventory model with shortage.

In this paper, the organization of the remaining content is outlined as follows: Section 2 provide the necessary notations and assumptions that will be used throughout the paper to develop proposed models. In Section 3, the development of mathematical models is presented. Section 4, demonstrated the effectiveness of proposed solution methodology by providing some numerical examples and sensitivity analysis. The paper concludes in Section 5.

2. Notations and Assumption

2.1 Notations

The Following Symbols Are Utilized During the Inventory Model Development

Symbol	Units	Description
c_0	\$/order	Ordering Cost
C	\$/Unit	Purchasing Cost
P	\$/Unit	Selling Price
C_h	\$/Unit/Unit time	Holding Cost
C_b	\$/Unit/Unit time	Shortage Cost
θ	$\theta \in (0,1)$	Deterioration Rate
P	Units/Unit time	Production Rate
$\tilde{D}$	Units/Unit time	Demand Rate

t_1	Unit time	Time When Stock Level Attins It's Maximum Level
t_2	Unit time	Time When Stock Level Touches Zero
t_3	Unit time	Time When the Inventory Level Achieves It's Maximum Shortage Level
T	Unit time	Replenishment Cycle
I (t)	Units	Inventory Level at Time t;$0 \le t \le T$
M	Unit time	The Retailer's Trade Credit Period Given By The Supplier
N	Unit time	Costumer Trade Credit Period Given By The Retailer
I_e	%/ Unit time	Interest Earned by The Retailer
I_p	%/ Unit time	Interest Paid by The Retailer
TC_i (S, R)	%/ Unit time	The Total Cost Where i=1,2…,5
Decision Variables		
S	Units	Order Quantity
R	Units	Shortage Level

2.2 Assumptions

The inventory model is based on the suppositions listed below:

1. Inventory system consists of a single deteriorating item.
2. By trapezoidal fuzzy number D = (D_1, D_2, D_3, D_4) the demand rate is characterized.
3. With a constant rate θ the on-hand inventory deteriorates, where $0 < \theta < 1$.
4. The deteriorated items are not repairable and replenishment is not possible for such item.
5. The planning horizon is infinite
6. Replenishment rate is instantaneous.
7. Stock out is permissible and unsatisfied demand is fully backlogged.
8. The trade credit policy to both retailer and customer.

3. Model Formulation

3.1 Crisp Model

We use Shaikh et al. (2021) model for crisp environment according to that model objective function for different cases are as follows:

Case 1: N < $M \le t_1 < t_2$

Problem 1
$$\left.\begin{aligned} & Minimize TC_1(S,R)=\frac{1}{T}\left[\begin{aligned} & A+c(S+R)+\frac{h}{\theta^2}\left\{De^{\theta(t_2-t_1)}+(P-D)e^{\theta t_1}+(Pt_1-Dt_2)\theta-P\right\} \\ & +\frac{S}{\theta^2}\left[(P-D)\left\{e^{\theta(T-t_3)}-\theta(T-t_3)-1\right\}+\frac{D\theta^2}{2}(t_3-t_2)^2\right] \\ & +cI_c\left[\begin{aligned} & \frac{P-D}{\theta^2}\left\{\theta(t_1-M)+e^{-\theta t_1}-e^{-\theta M}\right\}+ \\ & \frac{D}{\theta^2}\left\{e^{\theta(t_2-t_1)}-\theta(t_2-t_1)-1\right\} \end{aligned}\right]-\frac{PI_eD(M^2-N^2)}{2} \end{aligned}\right] \\ & Subject\ to\ N<M\le t_1<t_2 \end{aligned}\right\}$$

(3.1)

Case 2

Problem 2

$$\left.\begin{array}{l} Minimize TC_2(S,R)=\dfrac{1}{T}\left[\begin{array}{l} A+c(S+R)+\dfrac{h}{\theta^2}\left\{De^{\theta(t_2-t_1)}+(P-D)e^{\theta t_1}+(Pt_1-Dt_2)\theta-P\right\} \\ +\dfrac{S}{\theta^2}\left[(P-D)\left\{e^{\theta(T-t_3)}-\theta(T-t_3)-1\right\}+\dfrac{D\theta^2}{2}(t_3-t_2)^2\right] \\ +\dfrac{cI_C D}{\theta^2}\left\{e^{\theta(t_2-M)}-\theta(t_2-M)-1\right\} \\ -\dfrac{PI_e D(M^2-N^2)}{2} \end{array}\right] \\ Subject\ to\ N<t_1\le M\le t_2 \end{array}\right\} \quad (3.2)$$

Case 3: $t_1 \le N < M \le t_2$

Problem 3

$$\left.\begin{array}{l} \min TC_3(S,R)=\dfrac{1}{T}\left[\begin{array}{l} A+c(S+R)+\dfrac{h}{\theta^2}\left\{De^{\theta(t_2-t_1)}+(P-D)e^{-\theta t_1}+(Pt_1-Dt_2)\theta-P\right\} \\ +\dfrac{S}{\theta^2}\left[(P-D)\left\{e^{\theta(T-t_3)}-\theta(T-t_3)-1\right\}+\dfrac{D\theta^2}{2}(t_3-t_2)^2\right] \\ +\dfrac{cI_c D}{\theta^2}\left[e^{\theta(t_2-M)}-\theta(t_2-M)-1\right]-\dfrac{PI_e D(M^2-N^2)}{2} \end{array}\right] \\ subject\ to\ t_1\le N<M\le t_2 \end{array}\right\} \quad (3.3)$$

Case 4: $t_1 \le N < t_2 \le M$

Problem 4

$$\left.\begin{array}{l} \min TC_4(S,R)=\dfrac{1}{T}\left[\begin{array}{l} A+c(S+R)+\dfrac{h}{\theta^2}\left\{De^{\theta(t_2-t_1)}+(P-D)e^{-\theta t_1}+(Pt_1-Dt_2)\theta-P\right\} \\ +\dfrac{S}{\theta^2}\left[(P-D)\left\{e^{\theta(T-t_3)}-\theta(T-t_3)-1\right\}+\dfrac{D\theta^2}{2}(t_3-t_2)^2\right] \\ -\dfrac{PI_e D(M^2-N^2)}{2} \end{array}\right] \\ subject\ to\ t_1\le N<t_2\le M \end{array}\right\} \quad (3.4)$$

Case 5: $t_1 < t_2 \le N < M$

Problem 5

$$\left.\begin{array}{l} \min TC_5(S,R)=\dfrac{1}{T}\left[\begin{array}{l} A+c(S+R)+\dfrac{h}{\theta^2}\left\{De^{\theta(t_2-t_1)}+(P-D)e^{-\theta t_1}+(Pt_1-Dt_2)\theta-P\right\} \\ +\dfrac{S}{\theta^2}\left[(P-D)\left\{e^{\theta(T-t_3)}-\theta(T-t_3)-1\right\}+\dfrac{D\theta^2}{2}(t_3-t_2)^2\right] \\ -\dfrac{PI_e D(M^2-N^2)}{2} \end{array}\right] \\ subject\ to\ t_1<t_2\le N<M \end{array}\right\} \quad (3.5)$$

Where,

$$t_1=-\frac{1}{\theta}\left[\log(P-D-S\theta)-\log(P-D)\right] \quad (3.6)$$

$$t_2 = t_1 + \frac{1}{\theta}\left[\log(D+S\theta) - \log(D)\right] \tag{3.7}$$

$$t_3 = t_2 + \frac{R}{D} \tag{3.8}$$

$$T = t_3 + \frac{1}{\theta}\left[\log(P-D-R\theta) - \log(P-D)\right] \tag{3.9}$$

3.2 Fuzzy Model

Here demand rate is assumed to be a trapezoidal fuzzy number as $\tilde{D} = (D_{1,} D_{2,} D_{3,} D_4)$. The optimal fuzzy total cost of inventory system is then solved as $TC_i = (TC_{i1,} TC_{i2,} TC_{i3,} TC_{i4})$. The Fuzzy Total Inventory Cost (FTIC) is defuzzified using Graded Mean Representation Method. The result is

$$P(TC_i) = \frac{1}{6}(TC_{i1} + 2TC_{i2} + 2TC_{i3} + TC_{i4}) \tag{3.10}$$

Now, we formulate the above mentioned five cases in fuzzy sense.

Case 1: $N < M \le t_1 < t_2$

In this case the FTIC is given by $TC_1 = (TC_{11}, TC_{12}, TC_{13}, TC_{14})$ where TC_{1j} for j = 1,2,3,4 is defined as follows:

$$TC_{1j} = \frac{1}{T}\left[\begin{array}{l} A + c(S+R) + \frac{h}{\theta^2}\left[D_j e^{\theta(t_{2j}-t_{1j})} + (P-D_j)e^{-\theta t_{1j}} + (Pt_{1j} - D_j t_{2j})\theta - P\right] \\ + \frac{S}{\theta^2}\left[(P-D)\left\{e^{\theta(T_j - t_{3j})} - \theta(T_j - t_{3j}) - 1\right\} + \frac{D_j\theta^2}{2}\left(t_{3j} - t_{2j}\right)^2\right] \\ + cI_c\left[\begin{array}{l} \frac{P-D_j}{\theta^2}\left\{\theta(t_{1j} - M) + e^{-\theta t_{1j}} - e^{-\theta M}\right\} + \\ \frac{D_j}{\theta^2}\left\{e^{\theta(t_{2j}-t_{1j})} - \theta(t_{2j} - t_{1j}) - 1\right\} \end{array}\right] - \frac{PI_e D_j (M^2 - N^2)}{2} \end{array}\right] \tag{3.11}$$

Where

$$t_{1j} = -\frac{1}{\theta}\log\left(\frac{P - D_j - S\theta}{P - D_j}\right) \tag{3.12}$$

$$t_{2j} = t_{1j} + \frac{1}{\theta}\log\left(\frac{D_j + S\theta}{D_j}\right) \tag{3.13}$$

$$t_{3j} = t_{2j} + \frac{R}{D_j} \tag{3.14}$$

$$T_j = t_{3j} + \frac{1}{\theta}\log\left(\frac{P - D_j + R\theta}{P - D_j}\right) \tag{3.15}$$

And t_1, t_2, t_3 and T are given by

$$t_1 = \frac{1}{6}(t_{11} + 2t_{12} + 2t_{13} + t_{14}) \tag{3.16}$$

$$t_2 = \frac{1}{6}(t_{21} + 2t_{22} + 2t_{23} + t_{24}) \tag{3.17}$$

$$t_3 = \frac{1}{6}(t_{31} + 2t_{32} + 2t_{33} + t_{34}) \tag{3.18}$$

$$T = \frac{1}{6}(T_1 + 2T_2 + 2T_3 + T_4) \tag{3.19}$$

Now, the problem 1 reduced to

Problem 1

$$\left.\begin{array}{l} Minimize\, P(TC_1) \\ subject\ to\, N < M \le t_1 < t_2 \end{array}\right\} \tag{3.20}$$

Where $P(TC_1)$ can be obtained by putting i=1 in equation (3.10)

Case 2: $N < t_1 \le M \le t_2$

Here the fuzzy total inventory cost is $TC_2 = (TC_{21}, TC_{22}, TC_{23}, TC_{24})$ where TC_{2j} for j=1,2,3,4 is defined as follows:

$$TC_{2j} = \left[\begin{array}{l} A + c(S+R) + \frac{h}{\theta^2}\left[D_j e^{\theta(t_{2j}-t_{1j})} + (P-D_j)e^{-\theta t_{1j}} + (Pt_{1j} - D_j t_{2j})\theta - P\right] \\ +\frac{S}{\theta^2}\left[(P-D_j)\left\{e^{\theta(T_j - t_{3j})} - \theta\left(T_j - t_{3j}\right) - 1\right\} + \frac{D_j\theta^2}{2}(t_{3j} - t_{2j})^2\right] \\ +\frac{cI_C D_j}{\theta^2}\left[e^{\theta(t_{2j}-M)} - \theta(t_{2j} - M) - 1\right] - \frac{PI_e D_j (M^2 - N^2)}{2} \end{array}\right] \tag{3.21}$$

Where t_{1j}, t_{2j}, t_{3j} *and* T_j are given by equations (3.12) -(3.15) respectively and t_1, t_2, t_3 and T are calculated same as discussed in case 1

Hence the problem 2 reduced to

Problem 2:

$$\left.\begin{array}{l} Minimize\, P(TC_2) \\ subject\ to\, N < t_1 \le M \le t_2 \end{array}\right\} \tag{3.22}$$

One can find $P(TC_2)$ from equation (3.10) by putting i=2

Case 3: $t_1 \le N < M \le t_2$

The FTIC in this case is expressed as $TC_3 = (TC_{31}, TC_{32}, TC_{33}, TC_{34})$ where TC_{3j} for j=1,2,3,4 is defined as follows:

$$TC_{3j} = \frac{1}{T_j}\left[\begin{array}{l} A + c(S+R) + \frac{h}{\theta^2}\left[D_j e^{\theta(t_{2j}-t_{1j})} + (P-D_j)e^{-\theta t_{1j}} + (Pt_{1j} - D_j t_{2j})\theta - P\right] \\ \frac{S}{\theta^2}\left[(P-D_j)\left\{e^{\theta(T_j - t_{3j})} - \theta(T_j - t_{3j}) - 1\right\} + \frac{D_j\theta^2}{2}(t_{3j} - t_{2j})^2\right] \\ +\frac{cI_C D_j}{\theta^2}\left[e^{\theta(t_{2j}-M)} - \theta(t_{2j} - M) - 1\right] - \frac{PI_e D_j (M^2 - N^2)}{2} \end{array}\right] \tag{3.23}$$

Where t_{1j}, t_{2j}, t_{3j} and T_j are given by equation (3.12) -(3.15) respectively and t_1, t_2, t_3, and T can be obtained same as in case 1. Here problem (3) can be written as

Problem 3:

$$\left.\begin{array}{l} \text{Minimize } P\left(TC_3\right) \\ subject\ to\, t_1 \le N < M \le t_2 \end{array}\right\} \tag{3.24}$$

Where $P(TC_3)$ can be expressed by equation (3.10) for i=3.

Case 4: $t_1 \le N < t_2 \le M$

The FTIC in this instance can be written as

$$TC_{4j} = \frac{1}{T_j}\left[\begin{array}{l} A + c(S+R) + \frac{h}{\theta^2}\left[D_j e^{\theta(t_{2j}-t_{1j})} + (P-D_j)e^{-\theta t_{1j}} + (Pt_{1j} - D_j t_{2j})\theta - P\right] \\ + \frac{S}{\theta^2}\left[(P-D_j)\left\{e^{\theta(T_j - t_{3j})} - \theta(T_j - t_{3j}) - 1\right\} + \frac{D_j\theta^2}{2}(t_{3j}-t_{2j})^2\right] \\ - \frac{PI_e D_j(M^2 - N^2)}{2} \end{array}\right] \quad (3.25)$$

Where t_{1j}, t_{2j}, t_{3j} and T_j are given by equation (3.12) -(3.15) respectively and t_1, t_2, t_3, and T can be obtained same as in case 1. Here problem (4) can be written as

Problem 4

$$\left.\begin{array}{l} Minimize\ \ p(TC_4) \\ subject\ to\, t_1 \le N < t_2 \le M \end{array}\right\} \quad (3.26)$$

Where $P(TC_4)$ can be expressed by equation (3.10) for i=4

Case 5: $t_1 < t_2 \le N < M$

The FTIC in this instance can be written as

$$TC_{5j} = \frac{1}{T_j}\left[\begin{array}{l} A + c(S+R) + \frac{h}{\theta^2}\left[D_j e^{\theta(t_{2j}-t_{1j})} + (P-D_j)e^{-\theta t_{1j}} + (Pt_{1j} - D_j t_{2j})\theta - P\right] \\ + \frac{S}{\theta^2}\left[\left(P-D_j\right)\left\{e^{\theta(T_j - t_{3j})} - \theta\left(T_j - t_{3j}\right) - 1\right\} + \frac{D_j\theta^2}{2}\left(t_{3j}-t_{2j}\right)^2\right] \\ - \frac{PI_e D_j(M^2 - N^2)}{2} \end{array}\right] \quad (3.27)$$

Where t_{1j}, t_{2j}, t_{3j} and T_j are given by equation (3.12) -(3.15) respectively and t_1, t_2, t_3, and T can be obtained same as in case 1. Here problem (5) can be written as

Problem 5

$$\left.\begin{array}{l} Minimize\ P(TC_5) \\ subject\ to\, t_1 < t_2 \le N < M \end{array}\right\} \quad (3.28)$$

Where $P(TC_5)$ can be expressed by equation (3.10) for i =5

For optimality the necessary condition of objective function is $\frac{\partial}{\partial s}\left[P(TC_i)\right] = 0$ and $\frac{\partial}{\partial R}\left[P(TC_i)\right] = 0$ and sufficient condition are

$$\frac{\partial^2}{\partial S^2}\left[P(TC_i)\right] \ge 0, \frac{\partial^2}{\partial R^2}\left[P(TC_i)\right] \ge 0$$

$$and\left(\frac{\partial^2}{\partial S^2}\left[P(TC_i)\right]\right)\left(\frac{\partial^2}{\partial R^2}\left[P(TC_i)\right]\right) - \left(\frac{\partial^2}{\partial S \partial R}\left[P(TC_i)\right]\right)^2 \ge 0$$

As the corresponding optimization problem is greatly non-linear in nature, it is quite difficult to justify the optimality analytically, and hence the convexity is portrayed graphically.

4 Numerical Examples and Sensitivity Analysis

Numerical Example: There are five examples taken that are being presented and proved also to exemplifying and certifying the inventory model. Each of them describes a single case of inventory model. The data of the above discussed examples are given in table 1. The researchers have used the classical optimization technique in order to solve those five numerical examples. Table 2 consists the optimal solutions in fuzzy environment for all examples.

Table 1 Data for instances

Instance	1	2	3	4	5
A $/order	150	150	150	150	170
D1 units/year	430	450	410	430	520
D2 units/year	520	500	500	550	530
D3 units/year	530	510	570	570	600
D4 units/year	550	530	680	610	630
P units/year	1000	1000	1000	1000	800
p$/unit	45	45	45	45	45
c $/unit	20	20	20	20	35
h $/unit/year	15	15	15	15	20
ic %/year	0.15	0.15	0.15	0.15	0.12
Ie%/year	0.09	0.09	0.09	0.09	0.07
M year	0.08	0.1	0.1	0.3	0.3
N year	0.05	0.05	0.07	0.14	0.27
θ	0.05	0.05	0.05	0.05	0.05
cb $/unit/year	20	20	20	20	30

Table 2 the optimal solution for all instances in fuzzy environment

Instance	Case	S	R	t_1	t_2	t_3	T	$P(TC_i)$
1	$N < M \le t_1 < t_2$	45.1401	40.4381	0.0934	0.1818	0.2610	0.3444	5784.35
2	$N < t_1 \le M \le t_2$	45.2619	39.5764	0.0909	0.1815	0.2608	0.3400	5789.42
3	$t_1 \le N < M \le t_2$	44.2356	39.2699	0.0995	0.1835	0.2583	0.3462	5629.33
4	$t_1 \le N < t_2 \le M$	36.9518	27.1307	0.0827	0.1510	0.2013	0.2618	5444.78
5	$t_1 < t_2 \le N < M$	40.0735	24.9920	0.1802	0.2510	0.2952	0.4068	6451.33

Sensitivity Analysis: To study the effect of over/underestimation of input data over the optimal solution of initial stock level (S). Maximum shortage level (R), cycle length (T), the total cost P(TC_i), and time periods, t_1, t_2 and t_3, the researchers have used example 1. The analysis of the same is being executed by modifying (decreasing/increasing) the input data by -20% to +20%. The outcomes are calculated by varying one input data and sustaining the other one with original value. Table 3 exhibits the outcomes of the sensitivity analysis.

Table 3: sensitivity analysis for instance 1

parameter	% of variation	% of change in						
	in parameter	$P(TC_i^*)$	S^*	R^*	t_1^*	t_2^*	t_3^*	T^*
A	-20	-1.59	-11.25	-11.25	-11.28	-11.25	-11.25	-11.25
	-10	-0.77	-5.46	-5.46	-5.47	-5.46	-5.46	-5.46
	0	0	0	0	0	0	0	0
	10	0.73	5.18	5.18	5.19	5.18	5.18	5.17
	20	1.43	10.11	10.11	10.14	10.11	10.11	10.1
P	-20	-24.88	-12.74	-14.16	49.75	19.41	9.15	18.8

	-10	-11.01	-5.29	-6.18	19.63	7.53	3.39	7.01
	0	0	0	0	0	0	0	0
	10	8.96	3.93	4.71	-13.96	-5.27	-2.26	-4.91
	20	16.4	6.95	8.4	-24.45	-9.21	-3.9	-8.58
M	-20	0.2	1.43	1.43	1.44	1.43	1.43	1.43
	-10	0.11	0.76	0.76	0.76	0.76	0.76	0.76
	0	0	0	0	0	0	0	0
	10	-0.12	-0.84	-0.84	-0.85	-0.84	-0.84	-0.84
	20	-0.25	-1.78	-1.78	-1.78	-1.78	-1.78	-1.78
N	-20	-0.05	-0.33	-0.33	-0.33	-0.33	-0.33	-0.33
	-10	-0.02	-0.18	-0.17	-0.18	-0.18	-0.18	-0.17
	0	0	0	0	0	0	0	0
	10	0.03	0.19	0.19	0.19	0.19	0.19	0.19
	20	0.06	0.4	0.4	0.41	0.4	0.4	0.4

From table 3 one can observe the following points

1. As Demand p increases, total cost $P(TC_1)$, level of highest stock (S), level of shortage (R), the time of replenishment cycle (T), time to reach maximum stock level t_1, and time when inventory level reaches zero (t_2), increase, and time to reach maximum shortage level (t_3) decreases, which is an obvious result.
2. $P(TC_1)$, the total cost, S, highest stock level, R. maximum shortage level T, the replenishment cycle, time to reach maximum stock level, t_2 time when inventory level becomes zero, and t_3 the time for maximum shortage level increase, when the value of ordering cost A increases
3. If production rate P increases, the total cost $P(TC_1)$, highest stock level (S), and shortage level (R), increase but the replenishment cycle (T), time to reach maximum stock level (t_1). Time when inventory level is zero (t_2) and the time for maximum shortage level (t_3), decrease.
4. The total cost $P(TC_1)$, highest stock (S), shortage level (R), the replenishment cycle (T), time to reach maximum stock level (t_1), time when inventory level is zero (t_2), and the time for maximum shortage level (t_3) decrease as M increases.
5. The total cost $P(TC_1)$, highest stock (S), shortage level (R), the replenishment cycle (T) time to reach maximum stock level (t_1). Time when inventory level reaches zero (t_2). and the time for maximum shortage level (t_3) increase with the increase in the value of Ns

Conclusion: -

The present research focuses and has developed a production-inventory model for an item that deteriorates considering fully backlogged shortages and full two- level trade credit system. Here by trapezoidal fuzzy number, the demand rate is being characterized and the production rate is acknowledged and perpetual. Numerical examples are being used to assess the validation and effectiveness of the proposed inventory model. The results show the importance of the proposed inventory model to the retail industry in making decision under realistic scenarios.

The present research on inventory model has future scope of research extension focusing on following topics and areas of interests, such as by considering fully backlogging, inflation, partial trade credit policy, overtime production rate and, imperfect production processes.

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